An Unified Approach To Pseudo Scalar Meson Photoproductions Off Nucleons In The Quark Model

Zhenping Li[1]*, Hongxing Ye[1] and Minghui Lu[2]
[1]Physics Department, Peking University
Beijing 100871, P.R.China
[2] Institute of High Energy Physics, Beijing 100039, P.R.China

February 9, 2008

Abstract

An unified approach to the pseudo scalar meson (π, η, and K) photoproduction off nucleons are presented. It begins with the low energy QCD Lagrangian, and the resonances in the s- and u- channels are treated in the framework of the quark model. The duality hypothesis is imposed to limit the number of the t-channel exchanges. The CGLN amplitudes for each reaction are evaluated, which include both proton and neutron targets. The important role by the S-wave resonances in the second resonance region is discussed, it is particularly important for the K, η and η' photoproductions.

PACS numbers: 13.75.Gx, 13.40.Hq, 13.60.Le,12.40.Aa

*E-mail; ZPLI@ibm320h.phy.pku.edu.cn

1. Introduction

Recently, there have been considerable interests to study the meson photoproductions off nucleons. The data from ELSA in the kaon[1] and η[2] productions, from MAMI[3] and BATES[4] in the η production have been published. Further experiments have also been planned at the Continuous Electron Beam Accelerator Facility (CEBAF)[5] and other electron accelerator facilities, which will provide a complete set of data in π, K, and η photoproductions with much better energy and angular resolutions. This provides us a golden opportunity to study the structure of baryon resonances and a chellange to understand the reaction mechanism in terms of quantum Chromodynamics(QCD).

The theoretical investigations of meson photoproductions during the past 30 years have been concentrated in the isobaric models [6, 7, 8, 9, 10], in which the Feynman diagrammatic techniques are used so that the transition amplitudes are Lorentz invariant. The recent investigations by J-C David*et al.*[8] in the kaon photoproductions and by the RPI group[11] in the threshold region of the η photoproduction have been quite successful in describing the available data. Because the meson baryon interactions are treated in the phenomenological level, the isobaric models have no explicit connection with QCD, and the number of parameters in these models are generally related to the number of resonances that are included in calculations. Thus, it becomes increasingly important to investigate the reaction mechanism in term of quarks and gluons degrees of freedom. Such a program has its genesis with the early work of Copley, Karl and Obryk[12] and Feynman, Kisslinger and Ravndal[13] in the pion photoproduction, who provided the first clear evidence of underlying $SU(6)\otimes O(3)$ structure to the baryon spectrum. The following calculations and discussions with the consistent treatment of the relativistic effects[14] have not changed the conclusions of Refs. [12] and [13] significantly. These calculations in the framework of the quark models have been limited on the transition amplitudes that are extracted from the photoproduction data by the phenomenological models. The challenge is whether one could go one step further to confront the photoproduction data directly with the explicit quark and gluon degrees of freedom. Such a step is by no means trivial, since it requires that the transition amplitudes in the quark model have correct off-shell behavior, which are usually evaluated on shell. More importantly, it also requires that the model with explicit quark and gluon degrees of freedom give a good description of the contributions to the photoproductions from the non-resonant background, which are usually used to evaluate the contributions from s-channel resonances. The low energy theorem in the threshold pion photoproduction is a crucial test in this regard, which the non-resonant

contributions dominate in the threshold region. Our investigation[15] showed that the simple quark model is no longer sufficient to recover the low energy theorem, and one has to rely on low energy QCD Lagrangian so that the meson baryons interaction is invariant under the chiral transformation. Moreover, we found substantial contributions from the S-wave resonances in the second resonance region to the E_{0+} amplitudes of the neutral pion photoproductions. This shows the importance of the consistent treatment of both resonant and non-resonant contributions even in the threshold pion photoproductions. We have extended it to the kaon[16] and η[17] photoproductions by combining the low energy QCD Lagrangian and the quark model, and the initial results showed very good agreements between the theory and experimental data with far less parameters. The purpose of this paper is to present a comprehensive and unified approach to the meson photoproductions based on the low energy QCD Lagrangian. The duality hypothesis is also imposed to limit the number of the t-channel exchanges, which was not done in our previous investigation[16]. This reduces the number of free parameters even further, and in principle, there is only one parameter for each isospin channel, such as $\alpha_{\eta NN}$ in the η production or $\alpha_{KN\Lambda}$ and $\alpha_{KN\Sigma}$ for kaon productions.

The paper is organized as follows. In the section 2, the theoretical framework is established in meson photoproductions starting from the low energy QCD Lagrangian. The formalism in the chiral quark model is presented for the s and u channel resonances in section 3. We shall show how the CGLN amplitudes for the s- and u- channel resonances are derived in the quark model. Although our approach start with the low energy QCD Lagrangian, it could also be extended to the heavy pseudoscalar meson photoproduction, such as the η' production, as the meson quark coupling should also be either pseudo scalar or pseudo vector for the η'. In section 4, we discuss some important features of the quark model approach to meson photoproductions. In particular, the S-wave resonances in the second resonance region play an important role in the threshold region of K, η and η' photoproductions. Finally, the conclusion will be given in section 5.

2. The Model

To understand many of the successes of the nonrelativistic quark model, Manohar and Georgi proposed[18] the concept of chiral quarks, which is described by the effective Lagrangian

$$\mathcal{L} = \bar{\psi}\left[\gamma_\mu(i\partial^\mu + V^\mu + \gamma_5 A^\mu) - m\right]\psi + \ldots \tag{1}$$

where the vector and axial currents are

$$
\begin{aligned}
V_\mu &= \frac{1}{2}\left(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger\right), \\
A_\mu &= i\frac{1}{2}\left(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger\right), \\
\xi &= e^{i\pi/f}
\end{aligned}
\tag{2}
$$

f is a decay constant, the quark field ψ in the $SU(3)$ symmetry is

$$
\psi = \begin{pmatrix} \psi(u) \\ \psi(d) \\ \psi(s) \end{pmatrix}, \tag{3}
$$

and the field π is a $3 \otimes 3$ matrix;

$$
\pi = \begin{vmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{vmatrix}, \tag{4}
$$

in which the pseudoscalar mesons, π, K and η, are treated as Goldstone bosons so that the Lagrangian in Eq. 1 are invariant under the chiral transformation. Starting from this chiral Lagrangian, there are four components for the photoproductions of pseudoscalar mesons;

$$
\begin{aligned}
\mathcal{M}_{fi} = \langle N_f|H_{m,e}|N_i\rangle + \sum_j \Bigg\{ &\frac{\langle N_f|H_m|N_j\rangle\langle N_j|H_e|N_i\rangle}{E_i+\omega-E_j} \\
&+ \frac{\langle N_f|H_e|N_j\rangle\langle N_j|H_m|N_i\rangle}{E_i-\omega_m-E_j}\Bigg\} + \mathcal{M}_T
\end{aligned}
\tag{5}
$$

where $N_i(N_f)$ is the initial (final) state of the nucleon, and $\omega(\omega_m)$ represents the energy of incoming (outgoing) photons(mesons).

The first term in Eq. 5 is a seagull term, it is generated by the gauge transformation of the axial vector A_μ in the QCD Lagrangian. The corresponding quark-photon-meson vertex is given by

$$
H_{m,e} = \sum_j \frac{e_m}{f_m}\phi_m\bar{\psi}_j(q_f)\gamma_\mu^j\gamma_5^j\psi_j(q_i)A^\mu(\mathbf{k},\mathbf{r}_j), \tag{6}
$$

where $A^\mu(\mathbf{k},\mathbf{r}_j)$ and ϕ_m are the electromagnetic and meson fields respectively. Notice that the seagull term in Eq. 6 is proportional to the charge e_m of the outgoing mesons, it does not contribute to the productions of the charge

neutral mesons. As it will be shown later, this also leads to the forward peaking in differential cross sections for the charge meson production.

The second and the third terms are s- and u-channel contributions. There has been considerable information on the s- and u- channel resonances from π N scattering as well as the pion photoproductions, and the transition properties of these resonances, such as the electromagnetic transition as well as the meson decays, have been investigated extensively in the framework of the quark model. Our task in meson photoproductions off nucleons is to combine the electromagnetic transitions and the meson decays of these resonances together, in particular those evaluated in Ref. [19], and to express these transition amplitudes in terms of the standard CGLN amplitudes[20] so that the various experimental observables could be easily calculated[21]. This has been done for the proton target in the Kaon and the η production, we will present the complete evaluation of the CGLN amplitudes for the transitions from both proton and neutron targets to the resonances below 2 GeV, which corresponds to main quantum number in the harmonic oscillator wavefunction $n \leq 2$ in $SU(6) \otimes O(3)$ symmetry limit. The connection between the CGLN amplitudes for baryon resonances and the helicity amplitudes, $A_{1/2}$ and $A_{3/2}$, in the electromagnetic transition could be easily established, this has been discussed extensively in Ref. [16]. For those resonances above 2 GeV, there is little information on their properties, thus they are treated as degenerate so that the contributions from the resonances with quantum number n could be expressed in a compact form. Generally, the contributions from those resonances with the largest spin for a given quantum number n are the most important as the energy increases, this corresponds to spin $J = n + 1/2$ for the processes $\gamma N \to K\Lambda$ and $\gamma N \to \eta N$, and $J = n + 3/2$ for the reactions $\gamma N \to K\Sigma$ and $\gamma N \to \pi N$.

The contributions from the u-channel resonances are divided into two parts. The first part is the contributions from the resonances with the quantum number $n = 0$, which include the spin 1/2 resonances, such as the Λ, Σ and the nucleon, and the spin 3/2 resonances, such as the Σ^* in kaon productions and $\Delta(1232)$ resonance in π productions. Because of the mass splitting between spin 1/2 and 3/2 resonances with $n = 0$ are significant, they have to be treated separately. The transition amplitudes for these u-channel resonances will also be written in terms of the CGLN amplitudes, which will be given in next section. The second part comes from the resonances with the quantum number $n \geq 1$. As the contributions from the u-channel resonances are not sensitive to the precise mass positions, they are treated as degenerate as well, so that the contributions from these resonances could also be written in a compact form, which is also in terms of the CGLN amplitudes.

The last term in Eq. 5 is the t-channel charged meson exchange, it is proportional to the charge of outgoing mesons as well, thus it does not contribute to the process $\gamma N \to \eta N$. This term is required so that the total transition amplitude in Eq. 5 is invariant under the gauge transformation[22]. The other t-channel exchanges, such as the K^* and $K1$ exchanges in the kaon production, which played an important role in Ref. [7, 8], the ρ and ω exchanges in the η production are excluded due to the constraint of the duality hypothesis. This was not imposed in our early investigation[16] of the kaon photoproduction, in which the contribution from the K^* exchange was included. The duality hypothesis states that the inclusion of the t-channel exchanges may lead to a double counting problem if a complete set of resonances is introduced in the s- and u- channels. Dolen, Horn and Schmid[23] found that the t-channel ρ Reggie trajectories, which govern the asymptotic high energy behavior, can be extracted from s-channel N^* resonances of their low energy model. This constraint has been applied to the kaon photoproduction by Williams, Ji, and Cotanch[9] in their quantum hadrondynamic approach. The problem in their approach is that the minimum set of the s- and u-channel resonances was used in the calculation so that the model space is severely truncated from a complete set of resonances to a few resonances, in particular, the resonances with higher spins that are important in higher energies are neglected. Thus, the theory becomes an effective theory, and the duality constraint is less significant. On the other hand, the chiral quark model provides an ideal framework to apply the duality constraint since every resonance could be included in principle and without additional parameters. The explicit expression for the t-channel charged meson exchange is shown in the Appendix.

3. Formalism

Before we present our chiral quark model approach, it is very useful to review some basic kinematic feature of meson photoproductions. The differential cross section for meson photoproductions in the center of mass frame is

$$\frac{d\sigma^{c.m.}}{d\Omega} = \frac{\alpha_e \alpha_m (E_N + M_N)(E_f + M_f)}{4s(M_f + M_N)^2} \frac{|\mathbf{q}|}{|\mathbf{k}|} |\mathcal{M}'_{fi}|^2 \tag{7}$$

where the factor eg_A/f_m is removed from the transition matrix element $\mathcal{M}'_{fi}$ so that it becomes dimensionless, and $\sqrt{s} = E_N + \omega_\gamma = E_f + \omega_m$ is the total energy in the c.m. frame. The coupling constant α_m is related to the factor g_A/f_m by the generalized Goldberg-Treiman relation[24], however, the quark mass effects lead to about 30 percent deviation from the measured value, while the Goldberg-Trieman relation is accurate within 5 percent for the pion

couplings[25]. Therefore, the coupling α_m will be treated as a free parameter for K and η productions at present stage.

The transition matrix element $\mathcal{M}'_{fi}$ is expressed in terms of the CGLN amplitude,

$$\mathcal{M}'_{fi} = \mathbf{J} \cdot \boldsymbol{\epsilon} \tag{8}$$

where $\boldsymbol{\epsilon}$ is the polarization vector of incoming photons, and the current J is written as

$$\mathbf{J} = f_1 \boldsymbol{\sigma} + i f_2 \frac{(\boldsymbol{\sigma} \cdot \mathbf{q})(\mathbf{k} \times \boldsymbol{\sigma})}{|\mathbf{q}||\mathbf{k}|} + f_3 \frac{\boldsymbol{\sigma} \cdot \mathbf{k}}{|\mathbf{q}||\mathbf{k}|} \mathbf{q} + f_4 \frac{\boldsymbol{\sigma} \cdot \mathbf{q}}{\mathbf{q}^2} \mathbf{q} \tag{9}$$

in the center mass frame, where $\boldsymbol{\sigma}$ is the spin operator for the initial and final states with spin 1/2. Therefore, the differential cross section in terms of the CGLN amplitude is[21]

$$\begin{aligned} |\mathcal{M}'_{fi}|^2 = \quad & Re \quad \Big\{ |f_1|^2 + |f_2|^2 - 2\cos(\theta) f_2 f_1^* \\ + \quad & \frac{\sin^2(\theta)}{2} \Big[|f_3|^2 + |f_4|^2 + 2 f_4 f_1^* + 2 f_3 f_2^* + 2\cos(\theta) f_4 f_3^* \Big] \Big\} \end{aligned} \tag{10}$$

where θ is the angle between the incoming photon momentum $\mathbf{k}$ and outgoing meson momentum $\mathbf{q}$ in the center mass frame. The various polarization observables can also be expressed in terms of CGLN amplitudes, which can be found in Ref. [21].

Therefore, it is more convenient to express the transition amplitudes in the quark model in terms of the CGLN amplitudes, since the kinematics in this framework is well known. We start this procedure from the general quark-photon and quark meson interactions in the QCD Lagrangian in Eq. 1. By expanding the nonlinear field ξ in Eq. 2 in terms of the Goldstone boson fields π;

$$\xi = 1 + i\pi/f + \ldots, \tag{11}$$

we obtain the standard pseudovector coupling at the tree level;

$$H_m = \sum_j \frac{1}{f_m} \bar{\psi}_j \gamma_\mu^j \gamma_5^j \psi_j \partial^\mu \phi_m. \tag{12}$$

The electromagnetic coupling is

$$H_e = -\sum_j e_j \gamma_\mu^j A^\mu(\mathbf{k}, \mathbf{r}). \tag{13}$$

Because the baryon resonances in s- and u-channels are treated as three quark systems, the separation of the center of mass motion from the internal motions in the transition operators is crucial, in particular, to reproduce the model independent low energy theorems[15] in the threshold pion-photoproduction and in the Compton scattering, $\gamma N \to \gamma N$[26]. Thus, we take the same approach as that in Refs. [26] and [15] to evaluate the contributions from resonances in s- and u-channels. Replacing the spinor $\bar{\psi}$ by $\psi^\dagger$ so that the γ matrices are replaced by the matrix $\boldsymbol{\alpha}$, the matrix elements for the electromagnetic interaction H_e can be written as

$$\begin{aligned}\langle N_j|H_e|N_i\rangle &= \langle N_j|\sum_j e_j \boldsymbol{\alpha}_j \cdot \boldsymbol{\epsilon} e^{i\mathbf{k}\cdot\mathbf{r}_j}|N_i\rangle \\ &= i\langle N_j|[\hat{H}, \sum_j e_j \mathbf{r}_j \cdot \boldsymbol{\epsilon} e^{i\mathbf{k}\cdot\mathbf{r}_j}] - \sum_j e_j \mathbf{r}_j \cdot \boldsymbol{\epsilon}\boldsymbol{\alpha}_j \cdot \mathbf{k} e^{i\mathbf{k}\cdot\mathbf{r}_j}|N_i\rangle \\ &= i(E_j - E_i - \omega)\langle N_j|g_e|N_i\rangle + i\omega\langle N_j|h_e|N_i\rangle, \end{aligned} \tag{14}$$

where

$$\hat{H} = \sum_j (\boldsymbol{\alpha}_j \cdot \mathbf{p}_j + \beta_j m_j) + \sum_{i,j} V(\mathbf{r}_i - \mathbf{r_j}) \tag{15}$$

is the Hamiltonian for the composite system,

$$g_e = \sum_j e_j \mathbf{r}_j \cdot \boldsymbol{\epsilon} e^{i\mathbf{k}\cdot\mathbf{r}_j}, \tag{16}$$

$$h_e = \sum_j e_j \mathbf{r}_j \cdot \boldsymbol{\epsilon}(1 - \boldsymbol{\alpha}_j \cdot \hat{\mathbf{k}}) e^{i\mathbf{k}\cdot\mathbf{r}_j}, \tag{17}$$

and $\hat{\mathbf{k}} = \frac{\mathbf{k}}{\omega_\gamma}$. Similarly, we have

$$\langle N_f|H_e|N_j\rangle = i(E_f - E_j - \omega_\gamma)\langle N_f|g_e|N_j\rangle + i\omega_\gamma\langle N_f|h_e|N_j\rangle. \tag{18}$$

Therefore the second and the third terms in Eq. 5 can be written as

$$\begin{aligned}\mathcal{M}'_{23} = i\langle N_f|[g_e, H_m]|N_i\rangle + i\omega_\gamma \sum_j \Big\{ &\frac{\langle N_f|H_m|N_j\rangle\langle N_j|h_e|N_i\rangle}{E_i + \omega_\gamma - E_j} \\ &+ \frac{\langle N_f|h_e|N_j\rangle\langle N_j|H_m|N_i\rangle}{E_i - \omega_m - E_j} \Big\} \\ = \langle N_f|\mathcal{M}'_{seagul}|N_i\rangle + \langle N_f|\mathcal{M}_s|N_i\rangle &+ \langle N_f|\mathcal{M}_u|N_i\rangle \end{aligned} \tag{19}$$

where the first term could also be regarded as the seagull term, and the $\mathcal{M}_s(\mathcal{M}_u)$ corresponds to the s(u)-channel contributions. There are two very important consequences in this manipulation. First, the leading terms in the

low energy theorem of the threshold pion photoproduction are present only in the leading Born terms, which include the seagull term and the contributions from the nucleon in the s- and u-channel, while the resonance contributions are only present at higher order[15]. Second, the nonrelativistic expansion for h_e in Eq. 17 becomes[26, 15]

$$h_e = \sum_j \left[e_j \mathbf{r}_j \cdot \boldsymbol{\epsilon} - \frac{e_j}{2m_j} \boldsymbol{\sigma}_j \cdot (\boldsymbol{\epsilon} \times \hat{\mathbf{k}}) \right] e^{i\mathbf{k}\cdot\mathbf{r}_j}, \tag{20}$$

which h_e is only expanded to order $1/m_q$, and it has been shown[15] that the expansion to order $1/m_q$ is sufficient to reproduce the low energy theorem for the threshold pion-photoproductions[20]. The procedure from Eq. 14 to Eq. 20 is equivalent to the prescription in ref. [14], in which the effects of the binding potential is included in the transition operator so that the first term in Eq. 20 differs from $\frac{1}{m_q}\mathbf{p}_j \cdot \boldsymbol{\epsilon}$ used in Refs [12] and [19].

The corresponding meson-coupling is

$$H_m^{nr} = \sum_j \left\{ \frac{\omega_m}{E_f + M_f} \boldsymbol{\sigma}_j \cdot \mathbf{P}_f + \frac{\omega_m}{E_i + M_i} \boldsymbol{\sigma}_j \cdot \mathbf{P}_i - \boldsymbol{\sigma}_j \cdot \mathbf{q} \right.$$
$$\left. + \frac{\omega_m}{2\mu_q} \boldsymbol{\sigma}_j \cdot \mathbf{p}_j \right\} \frac{\hat{I}_j}{g_A} e^{-i\mathbf{q}\cdot\mathbf{r}_j} \tag{21}$$

where ω_m is the energy of the emitting mesons and $\hat{I}_j$ is an isospin operator. The factor $\frac{1}{\mu_q}$ in Eq. 21 is a reduced mass at the quark level, which equals $\frac{1}{\mu_q} = \frac{1}{m_s} + \frac{1}{m_q}$ for kaon productions and $\frac{1}{\mu_q} = \frac{2}{m_q}$ for η and π productions. The first three terms in Eq. 21 correspond to the center of mass motion, and the last term represents the internal transition. The constant g_A is related to the axial vector coupling, and defined as

$$\langle N_f | \sum_j \hat{I}_j \boldsymbol{\sigma}_j | N_i \rangle = g_A \langle N_f | \boldsymbol{\sigma} | N_i \rangle. \tag{22}$$

where $\boldsymbol{\sigma}$ is the total spin operator of the initial and final states with spin 1/2. The isospin operator $\hat{I}_j$ in Eq. 21 is

$$\hat{I}_j = \left\{ \begin{array}{rl} a_j^\dagger(s) a_j(u) & \text{for } K^+ \\ a_j^\dagger(s) a_j(d) & \text{for } K^0 \\ a_j^\dagger(d) a_j(u) & \text{for } \pi^+ \\ -\frac{1}{\sqrt{2}}(a_j^\dagger(u) a_j(u) - a_j^\dagger(d) a_j(d)) & \text{for } \pi^0 \\ 1 & \text{for } \eta \end{array} \right. , \tag{23}$$

where $a_j^\dagger(s)$ and $a_j(u)$ or $a_j(d)$ are the creation and annihilation operator for the strange and up or down quarks, and I_j is simply an unit operator for the η production so that the factor g_A is 1. The values of g_A are listed in Table 1 for each reaction in the $SU(6)$ symmetry limit.

3.1 The Seagull Term

The amplitudes for the seagull term is

$$\mathcal{M}_s = -F(\mathbf{k},\mathbf{q})e_m \left[1+\frac{\omega_m}{2}\left(\frac{1}{E_N+M_N}+\frac{1}{E_f+M_f}\right)\right]\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}. \tag{24}$$

where e_m is the charge of outgoing mesons, and the form factor is

$$F(\mathbf{k},\mathbf{q}) = exp\left(-\frac{(\mathbf{k}-\mathbf{q})^2}{6\alpha^2}\right) \tag{25}$$

in the harmonic oscillator basis, where α is the oscillator strength.

The seagull term in the chiral quark model is generated by the gauge transformation of the QCD Lagrangian in Eq. 1. It produces the leading term[15] in the low energy theorem in the threshold pion photoproduction. Therefore, it plays a dominant role in the meson photoproductions of nucleons in the low energy region. The form factor in Eq. 25 also makes this term peaked at the forward angle for finite **k** and **q**. This leads to an interesting prediction for meson photoproductions in the chiral quark model; the differential cross sections for the charged meson productions without contributions from isospin 3/2 resonances should be forward peaked above the threshold because of the dominance of seagull term in the low energy region. The data in the processes, $\gamma p \to K^+\Lambda$ and $\gamma p \to \eta p$, are consistent with this conclusion, in which the K^+ production is strongly forward peaked, while the η production does not exhibit the forward peaking at all. This feature is quite unique in the chiral quark model, it is a combination of the QCD Lagrangian and the integration of the spatial wavefunctions in the initial and final states, which does not exist in the traditional effective Lagrangian approaches at hadronic level.

3.2 The U-channel resonance contribution

We show the amplitudes for the u-channel Λ and Σ resonances in Kaon productions and for the u-channel nucleon in η and π productions in Appendix. The calculation of $\mathcal{M}_u$ in Eq. 19 for the excited states follows a procedure similar to that used in the Compton Scattering ($\gamma N \to \gamma N$)[26]. Replacing

the outgoing photon operator h_e in the Compton Scattering by H_m^{nr} in Eq. 21, then the $\mathcal{M}_u$ is

$$\mathcal{M}_u = \left(\mathcal{M}_u^3 + \mathcal{M}_u^2\right) e^{-\frac{\mathbf{k}^2+\mathbf{q}^2}{6\alpha^2}} \tag{26}$$

where

$$\begin{aligned} \frac{\mathcal{M}_u^3 g_A}{3} &= i\frac{e_3 I_3}{2m_q}\boldsymbol{\sigma}_3\cdot(\boldsymbol{\epsilon}\times\mathbf{k})\boldsymbol{\sigma}_3\cdot\mathbf{A}F^0(\frac{\mathbf{k}\cdot\mathbf{q}}{3\alpha^2}, P_f\cdot k) \\ -\frac{e_3 I_3}{6}\left[\frac{\omega_\gamma\omega_m}{\mu_q}\left(1+\frac{\omega_\gamma}{2m_q}\right)\boldsymbol{\sigma}_3\cdot\boldsymbol{\epsilon} + \frac{2\omega_\gamma}{\alpha^2}\boldsymbol{\sigma}_3\cdot\mathbf{A}\boldsymbol{\epsilon}\cdot\mathbf{q}\right] &F^1(\frac{\mathbf{k}\cdot\mathbf{q}}{3\alpha^2}, P_f\cdot k) \\ -\frac{\omega_\gamma\omega_m}{18\mu_q\alpha^2}e_3 I_3\boldsymbol{\sigma}_3\cdot\mathbf{k}\boldsymbol{\epsilon}\cdot\mathbf{q}&F^2(\frac{\mathbf{k}\cdot\mathbf{q}}{3\alpha^2}, P_f\cdot k), \end{aligned} \tag{27}$$

which corresponds to the outgoing meson and incoming photon absorbed and emitted by the same quark, and

$$\begin{aligned} \frac{\mathcal{M}_u^2 g_A}{6} &= i\frac{e_2 I_3}{2m_q}\boldsymbol{\sigma}_2\cdot(\boldsymbol{\epsilon}\times\mathbf{k})\boldsymbol{\sigma}_3\cdot\mathbf{A}F^0(-\frac{\mathbf{k}\cdot\mathbf{q}}{6\alpha^2}, P_f\cdot k) \\ +\frac{e_2 I_3}{12}&\left[\frac{\omega_\gamma\omega_m}{\mu_q}\left(\boldsymbol{\sigma}_3\cdot\boldsymbol{\epsilon} + \frac{1}{2m_q}\boldsymbol{\sigma}_2\cdot(\boldsymbol{\epsilon}\times\mathbf{k})\boldsymbol{\sigma}_3\cdot\mathbf{k}\right) + \frac{2\omega_\gamma}{\alpha^2}\boldsymbol{\sigma}_3\cdot\mathbf{A}\boldsymbol{\epsilon}\cdot\mathbf{q}\right] \\ \times F^1(-\frac{\mathbf{k}\cdot\mathbf{q}}{6\alpha^2}, P_f\cdot k) &- \frac{\omega_\gamma\omega_m}{72\mu_q\alpha^2}e_2 I_3\boldsymbol{\sigma}_3\cdot\mathbf{k}\boldsymbol{\epsilon}\cdot\mathbf{q}F^2(-\frac{\mathbf{k}\cdot\mathbf{q}}{6\alpha^2}, P_f\cdot k), \end{aligned} \tag{28}$$

which corresponds to the incoming photon and outgoing meson absorbed and emitted by different quarks. The vector $\mathbf{A}$ in Eqs. 27 and 28 are defined as

$$\mathbf{A} = -\omega_m\left(\frac{1}{E_N+M_N} + \frac{1}{E_f+M_f}\right)\mathbf{k} - \left(\omega_m\frac{1}{E_f+M_f}+1\right)\mathbf{q}. \tag{29}$$

Notice that the initial nucleon and the intermediate states have the c.m. momenta $-\mathbf{k}$ and $-\mathbf{k}-\mathbf{q}$ respectively. The function $F^l(x,y)$ in Eqs. 31 and 32 is the product of the spatial integral and the propagator for the excited states, it can be written as

$$F^l(x,y) = \sum_{n\geq l}\frac{M_n}{(n-l)!(y+n\delta M^2)}x^{n-l}, \tag{30}$$

where $n\delta M^2 = (M_n^2 - M_f^2)/2$ represents the mass difference between the ground state and excited states with the total excitation quantum number n in the harmonic oscillator basis.

The $\mathcal{M}_u^3$ in Eq. 27 and $\mathcal{M}_u^2$ in Eq. 28 are the u-channel operators at the quark level. They are the masteer equations for all pseudo scalar meson

photoproduction. To derive the amplitudes for a particular reaction, one has to transform Eqs. 27 and 28 into the more familiar CGLN amplitudes at the hadron level. We have

$$\begin{aligned}\frac{\mathcal{M}_u^3}{g_3^u} &= \frac{1}{2m}\left[ig_v\mathbf{A}\cdot(\boldsymbol{\epsilon}\times\mathbf{k})+\boldsymbol{\sigma}\cdot(\mathbf{A}\times(\boldsymbol{\epsilon}\times\mathbf{k}))\right]F^0(\frac{\mathbf{k}\cdot\mathbf{q}}{3\alpha^2},P_f\cdot k)\\ &-\frac{1}{6}\left[\frac{\omega_m\omega_\gamma}{\mu_q}\left(1+\frac{\omega_\gamma}{2m_q}\right)\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}+\frac{2\omega_\gamma}{\alpha^2}\boldsymbol{\sigma}\cdot\mathbf{A}\boldsymbol{\epsilon}\cdot\mathbf{q}\right]F^1(\frac{\mathbf{k}\cdot\mathbf{q}}{3\alpha^2},P_f\cdot k)\\ &-\frac{\omega_m\omega_\gamma}{18\alpha^2\mu_q}\boldsymbol{\sigma}\cdot\mathbf{k}\boldsymbol{\epsilon}\cdot\mathbf{q}F^2(\frac{\mathbf{k}\cdot\mathbf{q}}{3\alpha^2},P_f\cdot k),\end{aligned}\tag{31}$$

and

$$\begin{aligned}\frac{\mathcal{M}_u^2}{g_2^u} &= -\frac{1}{2m_q}\left[-ig_v'\mathbf{A}\cdot(\boldsymbol{\epsilon}\times\mathbf{k})+g_a'\boldsymbol{\sigma}\cdot(\mathbf{A}\times(\boldsymbol{\epsilon}\times\mathbf{k}))\right]F^0(\frac{-\mathbf{k}\cdot\mathbf{q}}{6\alpha^2},P_f\cdot k)\\ &+\frac{1}{12}\left[\frac{\omega_m\omega_\gamma}{\mu_q}\left(1+g_a'\frac{\omega_\gamma}{2m_q}\right)\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}+\frac{2\omega_\gamma}{\alpha^2}\boldsymbol{\sigma}\cdot\mathbf{A}\boldsymbol{\epsilon}\cdot\mathbf{q}\right]F^1(-\frac{\mathbf{k}\cdot\mathbf{q}}{6\alpha^2},P_f\cdot k)\\ &-\frac{\omega_m\omega_\gamma}{72\alpha^2\mu_q}\boldsymbol{\sigma}\cdot\mathbf{k}\boldsymbol{\epsilon}\cdot\mathbf{q}F^2(-\frac{\mathbf{k}\cdot\mathbf{q}}{6\alpha^2},P_f\cdot k).\end{aligned}\tag{32}$$

The various g-factors in Eqs. 31 and 32 are defined as

$$g_3^u = \langle N_f|\sum_j e_j\hat{I}_j\sigma_j^z|N_i\rangle/g_A,\tag{33}$$

$$g_2^u = \langle N_f|\sum_{i\neq j} e_j\hat{I}_i\sigma_i^z|N_i\rangle/g_A,\tag{34}$$

and

$$g_v = \langle N_f|\sum_j e_j\hat{I}_j|N_i\rangle/g_3^u g_A,\tag{35}$$

where g_A is given in Eq. 22. The factors g_v' and g_a' in Eq. 32 come from

$$\frac{1}{g_2^u g_A}\langle N_f|\sum_{i\neq j} e_j\hat{I}_i\boldsymbol{\sigma}_i\cdot\mathbf{A}\boldsymbol{\sigma}_j\cdot\mathbf{B}|N_i\rangle = \langle N_f|g_v'\mathbf{A}\cdot\mathbf{B}+ig_a'\boldsymbol{\sigma}\cdot(\mathbf{A}\times\mathbf{B})|N_i\rangle,\tag{36}$$

where $\mathbf{A}$ and $\mathbf{B}$ are vectors, and $\boldsymbol{\sigma}$ is the total spin operator for spin 1/2 baryons. Thus, we have

$$g_v' = \frac{1}{3g_2^u g_A}\langle N_f|\sum_{i\neq j} e_j\hat{I}_i\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j|N_i\rangle,\tag{37}$$

and

$$g_a' = \frac{1}{2g_2^u g_A}\langle N_f|\sum_{i\neq j} e_j \hat{I}_i(\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j)_z|N_i\rangle. \tag{38}$$

The numerical values of these g-factors in Eqs. 31 and 32 depend on the detialed structures of final state wavefunctions, and they are presented in Table 1 in the $SU(6)$ symmetry limit.

The first term in Eqs. 31 and 32 corresponds to the correlation between the magnetic transition and the c.m. motion of the kaon transition operator, it contributes to the leading Born term in the U-channel. The second term is the correlations among the internal and c.m. motions of the photon and meson transition operators, they only contribute to the transitions between the ground and $n \geq 1$ excited states in the harmonic oscillator basis. The last term in both equations represents the correlation of the internal motions between the photon and meson transition operators, which only contribute to the transition between the ground and $n \geq 2$ excited states. An interesting observation from these expressions is that the transition matrix elements $\mathcal{M}_u^3$ and $\mathcal{M}_u^2$ correspond to the incoming photons and outgoing kaons being absorbed and emitted by the same and different quarks, and they differ by a factor $\left(-\frac{1}{2}\right)^n$. Thus the transition matrix element $\mathcal{M}_u^3$ becomes dominant as the quantum number n increases.

Eqs. 31 and 32 can be summed up to any quantum number n, however, the excited states with large quantum number n become less significant for the u-channel resonance contributions. Thus, we only include the excited states with $n \leq 2$, which is the minimum number required for the contributions from every term in Eqs. 31 and 32. Physically, this corresponds to the average sum of the contributions from every resonance with the total excitation number $n = 1$ and 2. The orbital excited $n \geq 1$ resonances are treated as degenerate, since their contributions in the u-channel are much less sensitive to the detail structure of their masses than those in the s-channel. However, the contributions from $\Sigma^*(1385)$ and $\Delta(1232)$ to the K and π photoproductions should be separated from the Λ, Σ and nucleon for $n = 0$, as their masses differs significantly. The amplitude $\mathcal{M}_u$ for $n = 0$ is

$$\begin{aligned}\mathcal{M}_u^{n=0} = \frac{1}{2m_q}\frac{e^{-\frac{\mathbf{q}^2+\mathbf{k}^2}{6\alpha^2}}}{P_f\cdot k+\delta M^2/2}&[i(g_3^u g_v + g_2^u g_v')\mathbf{A}\cdot(\boldsymbol{\epsilon}\times\mathbf{k}) \\ &+(g_3^u - g_2^u g_a')\boldsymbol{\sigma}\cdot(\mathbf{A}\times(\boldsymbol{\epsilon}\times\mathbf{k}))]\end{aligned} \tag{39}$$

Eq. 39 represents the sum of the contributions from both spin 1/2 and 3/2 states in **56** multiplet, such as the Λ, Σ and Σ^* states in kaon photoproduction.

Thus, the contributions from spin 3/2 states such as the Σ^* and Δ can be obtained by subtracting the contributions of spin 1/2 intermediate states from the total $n=0$ amplitudes. The amplitude for spin 1/2 intermediate state is

$$\langle N_f|h_e|N(J=1/2)\rangle\langle N(J=1/2)|H_m|N_i\rangle = -\langle N_f|\mu\boldsymbol{\sigma}\cdot(\boldsymbol{\epsilon}\times\mathbf{k})\boldsymbol{\sigma}\cdot\mathbf{A}|N_i\rangle, \quad (40)$$

in which we only considered the contributions of the magnetic term in h_e. The magnetic moment μ in Eq. 40 is

$$\mu = \begin{cases} \mu_\Lambda + \frac{g_{K\Sigma N}}{g_{K\Lambda N}}\mu_{\Lambda\Sigma} & \text{for } \gamma N \to K\Lambda \\ \mu_{\Sigma^0} + \frac{g_{K\Lambda N}}{g_{K\Sigma N}}\mu_{\Lambda\Sigma} & \text{for } \gamma N \to K\Sigma^0 \\ \mu_{N_f} & \text{for other } \gamma N \to m_{0^{-+}} N_f \end{cases} . \quad (41)$$

We have

$$\mathcal{M}_u^{J=3/2} = \frac{1}{2m_q}\frac{e^{-\frac{\mathbf{q}^2+\mathbf{k}^2}{6\alpha^2}}}{P_f\cdot k+\delta M^2/2}[i(g_3^u g_v + g_2^u g_v')\mathbf{A}\cdot(\boldsymbol{\epsilon}\times\mathbf{k}) + (g_3^u - g_2^u g_a')\boldsymbol{\sigma}\cdot(\mathbf{A}\times(\boldsymbol{\epsilon}\times\mathbf{k})) - i\mu\boldsymbol{\sigma}\cdot(\boldsymbol{\epsilon}\times\mathbf{k})\boldsymbol{\sigma}\cdot\mathbf{A}] \quad (42)$$

We find that the general form of the CGLN amplitudes for the $J=3/2$ states with $n=0$ is

$$\mathcal{M}_u^{J=3/2} = \frac{M_f g_S e^{-\frac{\mathbf{q}^2+\mathbf{k}^2}{6\alpha^2}}}{M_N(P_f\cdot k+\delta M^2_{J=3/2}/2)}\left[i2\boldsymbol{\sigma}\cdot\mathbf{A}\boldsymbol{\sigma}\cdot(\boldsymbol{\epsilon}\times\mathbf{k}) + \boldsymbol{\sigma}\cdot(\mathbf{A}\times(\boldsymbol{\epsilon}\times\mathbf{k}))\right] \quad (43)$$

where $\delta M^2_{J=3/2} = M^2_{J=3/2} - M_f^2$. The factor g_S is also listed in Table 1 for each reaction. The structure in Eq. 43 is different from that of the CGLN amplitude for the spin 3/2 resonance in the s-channel; there are both M_1^- and M_1^+ components in Eq. 43, while the spin 3/2 resonance in the s-channel only has M_1^+ transition[21].

3.3 The S-channel resonance contribution

The amplitude of the nucleon pole term is presented in the Appendix. The S-channel resonance contributions comes from the second term in Eq. 19, the derivation of this term follows the analytic procedure for the Compton scattering in Ref. [26]. Replacing the outgoing photon vertex in Compton scattering in Ref. [26] by the meson transition operator in Eq. 21, we find that the general expression for the excited resonances in the s-channel can be written as

$$\mathcal{M}_R = \frac{2M_R}{s-M_R(M_R-i\Gamma(\mathbf{q}))}e^{-\frac{\mathbf{k}^2+\mathbf{q}^2}{6\alpha^2}}\mathcal{O}_R, \quad (44)$$

where $\sqrt{s} = E_i + \omega_\gamma = E_f + \omega_m$ is the total energy of the system, and $\mathcal{O}_R$ is determined by the structure of each resonance. Eq. 44 shows that there should be a form factor, $e^{-\frac{\mathbf{k}^2+\mathbf{q}^2}{6\alpha^2}}$ in the harmonic oscillator basis, even in the real photon limit. $\Gamma(\mathbf{q})$ in Eq. 44 is the total width of the resonance, and a function of the final state momentum $\mathbf{q}$. For a resonance decaying into a two-body final state with relative angular momentum l, the decay width $\Gamma(\mathbf{q})$ is

$$\Gamma(\mathbf{q}) = \Gamma_R \frac{\sqrt{s}}{M_R} \sum_i x_i \left(\frac{|\mathbf{q}_i|}{|\mathbf{q}_i^R|} \right)^{2l+1} \frac{D_l(\mathbf{q}_i)}{D_l(\mathbf{q}_i^R)}, \tag{45}$$

with

$$|\mathbf{q}_i^R| = \sqrt{\frac{(M_R^2 - M_N^2 + M_i^2)^2}{4M_R^2} - M_i^2}, \tag{46}$$

and

$$|\mathbf{q}_i| = \sqrt{\frac{(s - M_N^2 + M_i^2)^2}{4s} - M_i^2}, \tag{47}$$

where x_i is the branching ratio of the resonance decaying into a meson with mass M_i and a nucleon, and Γ_R is the total decay width of the S-channel resonance with the mass M_R. The function $D_l(\mathbf{q})$ in Eq. 45 is called fission barrier[28], and wavefunction dependent. For the meson transition operator in Eq.21, the $D_l(\mathbf{q})$ in the harmonic oscillator basis has the form[19]

$$D_l(\mathbf{q}) = exp\left(-\frac{\mathbf{q}^2}{3\alpha^2} \right), \tag{48}$$

which is independent of l. A similar formula used in I=1 $\pi\pi$ and p-wave $I = 1/2$ $K\pi$ scattering was found in excellent agreement with data in the ρ and K^* meson region[29]. In principle, the branching ratio x_i should be evaluated in the quark model. However, there are very large uncertainties in most quark model evaluation as the coupling constant, such as $\alpha_{\eta NN}$ and $\alpha_{K\Lambda N}$, are not well determined. Our results in the η and kaon photproductions could provide a guide for the future investigations, which in turn will determine the branching ratios x_i more precisely. Therefore, we simply set $x_\pi = x_\eta = 0.5$ for the resonance $S_{11}(1535)$, while $x_\pi = 1.0$ for the rest of the resonances as a first order approximation, as the resonance decays are dominated by the pion channels except the resonance $S_{11}(1535)$ whose branching ratio in ηN channel is around 50 percent. The results in η and kaon photoproductions suggest that they are not sensitive to the quantity x_i, as we know qualitatively that x_i is small in the ηN and KY channels except the case of $S_{11}(1535)$.

Our investigation in the η photoproduction[17] has shown that the momentum dependence of the decay width for the s-channel resonances is very important in extracting the properties of the resonance $S_{11}(1535)$ from the data in the threshold η production, it is also an important procedure to ensure the unitarity of the total transition amplitudes[11] approximately. This has not been taken into account in many calculations of the kaon and η production within the effective Lagrangian approach, which leads to a larger theoretical uncertainty that has not been fully investigated.

At the quark level, the operator $\mathcal{O}_R$ for a given n in the harmonic oscillator basis is

$$\mathcal{O}_n = \mathcal{O}_n^2 + \mathcal{O}_n^3 \tag{49}$$

where the amplitudes $\mathcal{O}_n^2$ and $\mathcal{O}_n^3$ have the same meaning as the amplitudes $\mathcal{M}_u^2$ and $\mathcal{M}_u^3$ in Eqs. 27 and 28. Following the same procedure used in the Compton Scattering[26], we have

$$\frac{\mathcal{O}_n^3 g_A}{3} = -i\frac{I_3 e_3}{2m_q}\boldsymbol{\sigma}_3\cdot\mathbf{A}\boldsymbol{\sigma}_3\cdot(\boldsymbol{\epsilon}\times\mathbf{k})\frac{1}{n!}\left(\frac{\mathbf{k}\cdot\mathbf{q}}{3\alpha^2}\right)^n$$
$$+\frac{e_3 I_3}{6}\left[\frac{\omega_\gamma\omega_m}{\mu_q}\left(1+\frac{\omega_\gamma}{2m_q}\right)\boldsymbol{\sigma}_3\cdot\boldsymbol{\epsilon}+\frac{2\omega_\gamma}{\alpha^2}\boldsymbol{\sigma}_3\cdot\mathbf{A}\boldsymbol{\epsilon}\cdot\mathbf{q}\right]\frac{1}{(n-1)!}\left(\frac{\mathbf{k}\cdot\mathbf{q}}{3\alpha^2}\right)^{n-1}$$
$$+\frac{\omega_\gamma\omega_m}{18\mu_q\alpha^2}e_3 I_3\boldsymbol{\sigma}_3\cdot\mathbf{k}\boldsymbol{\epsilon}\cdot\mathbf{q}\frac{1}{(n-2)!}\left(\frac{\mathbf{k}\cdot\mathbf{q}}{3\alpha^2}\right)^{n-2}, \tag{50}$$

and

$$\frac{\mathcal{O}_n^2 g_A}{6} = -i\frac{e_2 I_3}{2m_q}\boldsymbol{\sigma}_2\cdot(\boldsymbol{\epsilon}\times\mathbf{k})\boldsymbol{\sigma}_3\cdot\mathbf{A}\frac{1}{n!}\left(\frac{-\mathbf{k}\cdot\mathbf{q}}{6\alpha^2}\right)^n$$
$$-\frac{e_2 I_3}{12}\left[\frac{\omega_\gamma\omega_m}{\mu_q}\left(\boldsymbol{\sigma}_3\cdot\boldsymbol{\epsilon}+\frac{1}{2m_q}\boldsymbol{\sigma}_2\cdot(\boldsymbol{\epsilon}\times\mathbf{k})\boldsymbol{\sigma}_3\cdot\mathbf{k}\right)+\frac{2\omega_\gamma}{\alpha^2}\boldsymbol{\sigma}_3\cdot\mathbf{A}\boldsymbol{\epsilon}\cdot\mathbf{q}\right]$$
$$\times\frac{1}{(n-1)!}\left(\frac{-\mathbf{k}\cdot\mathbf{q}}{6\alpha^2}\right)^{n-1}+\frac{\omega_\gamma\omega_m}{72\mu_q\alpha^2}e_2 I_3\boldsymbol{\sigma}_3\cdot\mathbf{k}\boldsymbol{\epsilon}\cdot\mathbf{q}\frac{1}{(n-2)!}\left(\frac{-\mathbf{k}\cdot\mathbf{q}}{6\alpha^2}\right)^{n-2}, \tag{51}$$

where the vector $\mathbf{A}$ for the s-channel is

$$\mathbf{A} = -\left(\frac{\omega_m}{E_f+M_f}+1\right)\mathbf{q}. \tag{52}$$

One can transform Eqs. 50 and 51 into more familiar CGLN amplitudes, and we find

$$\frac{\mathcal{O}_n^3}{g_3^s} = -\frac{1}{2m_q}\left[ig_v\mathbf{A}\cdot(\boldsymbol{\epsilon}\times\mathbf{k})-\boldsymbol{\sigma}\cdot(\mathbf{A}\times(\boldsymbol{\epsilon}\times\mathbf{k}))\right]\frac{1}{n!}\left(\frac{\mathbf{k}\cdot\mathbf{q}}{3\alpha^2}\right)^n$$

$$+\frac{1}{6}\left[\frac{\omega_m\omega_\gamma}{\mu_q}\left(1+\frac{\omega_\gamma}{2m_q}\right)\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}+\frac{2\omega_\gamma}{\alpha^2}\boldsymbol{\sigma}\cdot\mathbf{A}\boldsymbol{\epsilon}\cdot\mathbf{q}\right]\frac{1}{(n-1)!}\left(\frac{\mathbf{k}\cdot\mathbf{q}}{3\alpha^2}\right)^{n-1}$$
$$+\frac{\omega_m\omega_\gamma}{18\alpha^2\mu}\boldsymbol{\sigma}\cdot\mathbf{k}\boldsymbol{\epsilon}\cdot\mathbf{q}\frac{1}{(n-2)!}\left(\frac{\mathbf{k}\cdot\mathbf{q}}{3\alpha^2}\right)^{n-2} \tag{53}$$

and

$$\frac{\mathcal{O}_n^2}{g_2^u}=\frac{1}{2m_q}\left[-ig_v'\mathbf{A}\cdot(\boldsymbol{\epsilon}\times\mathbf{k})+g_a'\boldsymbol{\sigma}\cdot(\mathbf{A}\times(\boldsymbol{\epsilon}\times\mathbf{k})\right]\frac{1}{n!}\left(\frac{-\mathbf{k}\cdot\mathbf{q}}{6\alpha^2}\right)^n$$
$$-\frac{1}{12}\left[\frac{\omega_m\omega_\gamma}{\mu_q}\left(1+g_a'\frac{\omega_\gamma}{2m_q}\right)\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}+\frac{2\omega_\gamma}{\alpha^2}\boldsymbol{\sigma}\cdot\mathbf{A}\boldsymbol{\epsilon}\cdot\mathbf{q}\right]\frac{1}{(n-1)!}\left(\frac{-\mathbf{k}\cdot\mathbf{q}}{6\alpha^2}\right)^{n-1}$$
$$+\frac{\omega_m\omega_\gamma}{72\alpha^2\mu}\boldsymbol{\sigma}\cdot\mathbf{k}\boldsymbol{\epsilon}\cdot\mathbf{q}\frac{1}{(n-2)!}\left(\frac{-\mathbf{k}\cdot\mathbf{q}}{6\alpha^2}\right)^{n-2}. \tag{54}$$

where the g-factors in Eqs. 53 and 54 are defined in Eqs. 33-38 and given in Table 1, and

$$g_3^s=\langle N_f|\sum_j e_j\hat{I}_j\sigma_j^z|N_i\rangle/g_A=e_m+g_3^u, \tag{55}$$

where e_m is the charge of the outgoing mesons. Thus, the operator $\mathcal{O}_R$ in Eq. 44 has a general structure,

$$\mathcal{O}_R=g_RA\left[f_1^R\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}+if_2^R(\boldsymbol{\sigma}\cdot\mathbf{q})\boldsymbol{\sigma}\cdot(\mathbf{k}\times\boldsymbol{\epsilon})+f_3^R\boldsymbol{\sigma}\cdot\mathbf{k}\boldsymbol{\epsilon}\cdot\mathbf{q}+f_4^R\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\epsilon}\cdot\mathbf{q}\right], \tag{56}$$

for the pseudoscalar meson photoproductions, where g_R is an isospin factor, A the meson decay amplitude, and f_i^R $(i=1\ldots4)$ is the photon transition amplitude. The factor g_R and the meson decay amplitude A in Eq. 56 are determined by the matrix elements $\langle N_f|H_m|N_j\rangle$ in Eq. 5; the factor g_R represents the transition in the spin-flavor space, and the amplitude A is the integral of the spatial wavefunctions.

We shall discuss briefly how the CGLN amplitudes for each resonance with $n=1$ could be extracted from Eq. 53 and 54, as the CGLN amplitudes for $J=3/2$ resonances with n=0 follow the same procedure as that in the u-channel. Since the amplitude $\mathcal{O}_{n=1}$ represents the sum of all resonances with $n=1$, we start with the reaction $\gamma p\to K^+\Lambda$, in which the isospin 3/2 does not contribute. Moreover, the contributions from the states with quantum number $N(^4P_M)$ vanish as well due to Moorhouse selection rule[31] for the electromagnetic transition h_e in Eq. 20. Thus, only the resonances with $N(^2P_M)$ contribute to the reaction $\gamma p\to K^+\Lambda$. Substitute the g-factors

for the reaction $\gamma p \to K^+\Lambda$ into Eqs. 53 and 54, we have

$$\mathcal{O}_{n=1} = -\frac{1}{12m_q}\left[i\mathbf{A}\cdot(\boldsymbol{\epsilon}\times\mathbf{k}) - \boldsymbol{\sigma}\cdot(\mathbf{A}\times(\boldsymbol{\epsilon}\times\mathbf{k}))\right]\frac{\mathbf{k}\cdot\mathbf{q}}{\alpha^2} + \frac{1}{12}\left[\frac{\omega_m\omega_\gamma}{\mu}\left(1+\frac{\omega_\gamma}{2m_q}\right)\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon} + \frac{2\omega_\gamma}{\alpha^2}\boldsymbol{\sigma}\cdot\mathbf{A}\boldsymbol{\epsilon}\cdot\mathbf{q}\right], \quad (57)$$

in which only S and D partial waves are present. Rewrite the quantity $\mathcal{O}_{n=1}$ into S and D waves, we find

$$\mathcal{O}_{n=1}(S\ wave) = \frac{\omega_\gamma}{12}\left(1+\frac{\omega_\gamma}{2m_q}\right)\left(\frac{\omega_m}{\mu_q}+\frac{2}{3}\frac{\mathbf{A}\cdot\mathbf{q}}{\alpha^2}\right)\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}, \quad (58)$$

and

$$\mathcal{O}_{n=1}(D\ wave) = \frac{-i}{12m_q}\boldsymbol{\sigma}\cdot\mathbf{A}\boldsymbol{\sigma}\cdot(\boldsymbol{\epsilon}\times\mathbf{k}) - \frac{\omega_\gamma}{18}\frac{\mathbf{A}\cdot\mathbf{q}}{\alpha^2}\left(1+\frac{\omega_\gamma}{2m_q}\right)\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon} + \frac{1}{6}\frac{\omega_\gamma}{\alpha^2}\boldsymbol{\sigma}\cdot\mathbf{A}\boldsymbol{\epsilon}\cdot\mathbf{q}. \quad (59)$$

Thus, the $\mathcal{O}_{n=1}(S\ wave)$ in Eq. 58 represents the CGLN amplitude for the resonance S_{11} with quantum number $N(^2P_M)\frac{1}{2}^-$, while $\mathcal{O}_{n=1}(D\ wave)$ is the CGLN amplitude for the resonance D_{13} with quantum number $N(^2P_M)\frac{3}{2}^-$, as only the S_{11} resonance with $N(^2P_M)\frac{1}{2}^-$ and the D_{13} resonance with $N(^2P_M)\frac{3}{2}^-$ contribute to $\gamma p \to K^+\Lambda$. The quantity $\left(\frac{\omega_m}{\mu_q}+\frac{2}{3}\frac{\mathbf{A}\cdot\mathbf{q}}{\alpha^2}\right)$ in Eq. 58 corresponds to the meson transition amplitude A in Eq. 56, while the meson transition amplitude A for the $D-wave$ resonance is $\frac{|\mathbf{A}|}{|\mathbf{q}|}$. The amplitudes A for the S and D waves have the same expressions as those in Table 1 of Ref. [19] with $g-\frac{1}{3}h=\frac{|\mathbf{A}|}{|\mathbf{q}|}$, and $h=\frac{\omega_m}{2\mu_q}$. Note that $\mathbf{A}$ has a negative sign, this is consistent with the fitted values for $g-\frac{1}{3}h$ and h in Ref. [19]. The quantity $\frac{\omega_\gamma}{12}\left(1+\frac{\omega_\gamma}{2m_q}\right)$ in Eq. 58 represents the photon transition amplitude f_i^R. It is proportional to the helicity amplitude $A^p_{\frac{1}{2}}$ for the state $N(^2P_M)\frac{1}{2}^-$ for the h_e in Eq. 20[30], as the CGLN amplitude for S-wave resonances is simply a product of photon and meson transition amplitudes.

Similarly, we find that the CGLN amplitude for the S_{31} resonance in the reaction $\gamma p \to K^+\Sigma^0$ is

$$\mathcal{O}_{n=1}(S\ wave) = \frac{\omega_\gamma}{6}\left(1-\frac{\omega_\gamma}{6m_q}\right)\left(\frac{\omega_m}{\mu_q}+\frac{2}{3}\frac{\mathbf{A}\cdot\mathbf{q}}{\alpha^2}\right)\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}. \quad (60)$$

This shows that the amplitude A is the same for both reactions, $\gamma p \to K^+\Lambda$ and $\gamma p \to K^+\Sigma^0$. In fact, it has been shown in Ref. [19] that the meson

transition amplitude A is independent of not only a particular reaction but also $SU(6)$ symmetry so that the resonances belonging to (56) and (70) multiplets with the same angular momentum $L = 2$ are governed by the same meson decay amplitude A. In other words the amplitude A in Eq. 56 is universal for the pseudoscalar meson decay processes. Thus, we present the amplitude A in the simple harmonic oscillator basis in Table 2, in which the amplitude A depends on the total excitation n and the orbital angular momentum L. The relative angular momentum of the final decay products is expressed in terms of the partial wave language in Table 2, which the S, P, D and F waves denote the relative angular momentum 0, 1, 2 and 3 between the final decay products.

Thus, the advantages of Eq. 56 are that only the factor g_R is determined by a particular reaction, while the amplitude A is universal, and the photon transition amplitudes f_i^R only depend on the initial proton and neutron targets. We show the photon transition amplitudes f_i^R for each resonance with $n \leq 2$ in Table 3 for the proton target and Table 4 for the neutron target. They are usually expressed in terms of helicity amplitudes, $A_{1/2}$ and $A_{3/2}$, and the connection between the two representations can be established, which has been discussed extensively in Refs. [16] and [17] for proton targets. Here we discuss some important features of the CGLN amplitudes for neutron targets and their relation to those for proton targets. A very important example is the contributions from the resonances belonging to $(70,^4N)$ representation for neutron targets, of which the transition amplitudes for proton targets are zero due to the Moorhouse selection rule[31] if one uses the nonrelativistic transition operator in Eq. 21. There are three important negative parity baryons that belong to the $(70,^4N)$ multiplet in the naive quark model, which correspond to $S_{11}(1650)$, $D_{13}(1700)$ and $D_{15}(1675)$. In particular, the contributions from the resonance $D_{15}(1675)$ are quite large. In general, the constraints on the phototransition in terms of the helicity amplitudes, $A_{\frac{1}{2}}$ and $A_{\frac{3}{2}}$ in the $SU(6)\otimes O(3)$ symmetry limit can also be applied to the corresponding CGLN amplitudes. For example, the helicity amplitude $A_{\frac{3}{2}}$ for the resonance $D_{13}(1520)$ classified as $N(^2P_M)\frac{3}{2}^-$ in the quark model has a simple relation[14, 19];

$$A^p_{\frac{3}{2}}(D_{13}(1520)) = -A^n_{\frac{3}{2}}(D_{13}(1520)), \tag{61}$$

and we find the same relation for the corresponding CGLN amplitudes f_4

$$f_4^p(D_{13}(1520)) = -f_4^n(D_{13}(1520)) \tag{62}$$

between protons and neutrons.

The CGLN amplitudes for three S-wave resonances show a more explicit connection with the corresponding helicity amplitudes. Because only f_1^R is present for the S-wave resonances, it is proportional to the helicity amplitude $A_{1/2}$. Thus, we have

$$\frac{f_1^R(\gamma p \to S_{11})}{f_1^R(\gamma n \to S_{11})} = \frac{A^p_{1/2}(S_{11})}{A^n_{1/2}(S_{11})}, \tag{63}$$

and the comparison between the CGLN amplitudes in Table 4 and 5 and the corresponding helicity amplitudes in Ref. [14] show that this is indeed the case.

For the excited positive parity baryon resonances, the helicity amplitude $A^n_{\frac{3}{2}}$ vanishes for the states $N(^2D_s)$, and corresponding CGLN amplitude f_4^n is zero for these states as well. The ratio of the helicity amplitudes $A_{\frac{1}{2}}$ between the proton and the neutron targets for the resonance $P_{11}(1440)$ is the same as the ratio of the CGLN amplitude f_2, which corresponds to the M_1^- transition according to the multipole decomposition of the CGLN amplitude[21]. There are also contributions from the states $N(^4D_M)$, which are in the same $SU(6)$ representation as the states $N(^4P_M)$ so that the Moorhouse selection rule is also true for these states. However, we find that only the CGLN amplitudes for the state $F_{17}(1990)$ is relatively strong, and there is little evidence for other resonances below 2 GeV. Therefore, only the contribution from $F_{17}(1990)$ will be taken into account in our calculation. The CGLN amplitudes for resonances P_{33} belonging to the 56 representations satisfy the relation

$$\frac{f_1^R}{\mathbf{q}\cdot\mathbf{k}} = -f_3^R = \frac{3}{2}f_2^R. \tag{64}$$

According to the multipole decomposition of the CGLN amplitudes[21], Eq. 62 corresponds to the M_1^+ transition which also leads to the relation[14, 19]

$$A_{1/2} = \frac{1}{\sqrt{3}}A_{3/2} \tag{65}$$

between the two helicity amplitudes. This is certainly true for the resonances $P_{33}(1232)$ and $P_{33}(1600)$ in the symmetry limit.

We present the quark model results of the g_R-factor for the pseudoscalar meson photoproductions in Table 5. It represents the relative strength and phases of the contributions from different resonances comparing to the contributions from the nucleon which belongs to the $(56, N)$ multiplet in $SU(6)$

symmetry. Notice that for a given $SU(3)$ representation, the factor g_R is determined by the C-G coefficient in the isospin coupling between the meson and the final baryon state,

$$g_R \propto \langle I_m, I_m^z, I_f, I_f^z | I_R, I_m^z + I_f^z \rangle / g_A, \tag{66}$$

where I_m^z and I_m are the isospin for the outgoing mesons, I_f and I_f^z are the isospin quantum numbers for final baryons, and I_R is the isospin of s-channel resonances. Thus, we have the relation

$$\frac{g_R(\gamma p \to K^0\Sigma^+)}{g_R(\gamma n \to K^+\Sigma^-)} = \frac{\langle \frac{1}{2}, -\frac{1}{2}, 1, 1 | I_R, \frac{1}{2} \rangle g_A(\gamma n \to K^+\Sigma^-)}{\langle \frac{1}{2}, \frac{1}{2}, 1, -1 | I_R, -\frac{1}{2} \rangle g_A(\gamma p \to K^0\Sigma^+)} = (-1)^{I_R - \frac{1}{2}}, \tag{67}$$

in which the additional minus sign comes from the ratio of the g_A. Results in Table 2 show that the relation in Eq. 67 is indeed satisfied. Therefore, the reaction $\gamma p \to K^0\Sigma^+$ could be regarded as a mirror of the reaction $\gamma n \to K^+\Sigma^-$ in the isospin space. The similar relations are also true for the reactions $\gamma p \to K^+\Lambda$ and $\gamma n \to K^0\Lambda$, $\gamma p \to K^+\Sigma^0$ and $\gamma n \to K^0\Sigma^+$, $\gamma p \to \pi^+ n$ and $\gamma n \to \pi^- p$, and $\gamma p \to \pi^0 p$ and $\gamma n \to \pi^0 n$, in which the relation in Eq. 67 is satisfied[1]. Thus, the coefficient g_R for the processes with the neutron target can be deduced from that of the proton target according to their isospin couplings, and this result seems to be more general than the $SU(6) \otimes O(3)$ basis used here. This also gives us an important constraint in predicting the reaction of neutron targets from the proton target results.

If one intends to calculate the reaction beyond 2 GeV in the center of mass frame, the higher resonances with quantum number $N = 3$ and $N = 4$ must be included. There are only a few resonances around 2 GeV that can be in principle classified as $N = 3$ resonances, in particular the resonances $S_{31}(1900)$ and $D_{35}(1930)$. However, we do not expect these resonances contribute significantly since they are lower partial wave resonances. Instead, we adopt an approach that treats the resonances for $N \geq 2$ as degenerate, the sum of the transition amplitudes from these resonances can be obtained through the approach in Ref. [26]. in Eqs. 53 and 54 becomes in the s-channel. Generally, the resonances with larger quantum number N become important as the energy increases. Note that the amplitude $\mathcal{O}_n^2$ generally differs from the amplitude $\mathcal{O}_n^3$ by a factor of $\left(-\frac{1}{2}\right)^n$, this shows that the process that the incoming photon and outgoing meson are absorbed and emitted by the same quark becomes more and more dominant as the energy increases. Furthermore, the resonances

[1]Except the states $(70,^4 N)$, of which the photon transition amplitudes vanish for the proton target

with partial wave $L = N$ become dominant, of which the isospin is 1/2 for $\gamma N \to K\Lambda$ and $\gamma N \to \eta N$ and 3/2 for $\gamma N \to K\Sigma$ and $\gamma N \to \pi N$. Thus, we could use the mass and decay width of the high spin states in Eq. 48; the resonance $G_{17}(2190)$ for the $n = 3$ states and the resonance $H_{19}(2220)$ for the $n = 4$ states in $\gamma N \to K\Lambda$ and $\gamma N \to \eta N$, and the resonance $G_{37}(2200)$ for the $n = 3$ states and the resonance $H_{3,11}(2420)$ for n=4 states in $\gamma p \to K\Sigma$. Indeed, only the couplings for the high spin states are strong enough to be seen experimentally, and this is consistent with the conclusions of the quark model.

4 Discussions

Eq. 56 establishes the connection between the transition amplitudes of the s-channel resonances and their underlying spin flavor structure. The relative strength and phase for each s-channel resonance are determined by the $SU(6) \otimes O(3)$ symmetry so that no additional parameters are required. Therefore, there are some important features of the s-channel resonances in meson photoproductions that can be discussed without numerical evaluation. Here we highlight some of them.

First, the S-wave resonances play very important roles in the threshold region, of which the transition amplitudes are determined by E_0^+ transition. This is particular true for the kaon and η production, in which masses of these S-wave resonances are sandwiched between their threshold energies. Moreover, the effects of the S-wave resonances are enhanced for the neutral meson production, since the seagull term that dominates in this region does not contribute. This has been widely recognized in the η photoproduction[17, 11], and their contributions to the threshold pion-photoproductions have been discussed recently[15]. The same is true for the kaon-photoproductions as well. Therefore, the kaon and η productions in the threshold region provides very important probe to the structure of these s-wave resonances. In Ref. [32], we showed that the kaon productions experiments may provide us information on the existence of a quasi bound $K\Lambda$ or $K\Sigma$ state, which has the same quantum number as the the resonance S_{11}. This will help us to understand the puzzle that the decay into ηN is enhanced for the the resonance $S_{11}(1535)$ and suppressed for the resonance $S_{11}(1650)$.

Second, assuming the η' quark coupling is either pseudo scalar or pseudo vector, one could extend this approach from η to η' photoproduction. An interesting prediction[33] from the quark model emerges for the η' photoproduction; the threshold behavior of the η' photoproduction is dominated by the off-shell contributions from the s-wave resonances in the second resonance

region, which can be tested in the future CEBAF experiments[35]. This can be understood by the relative strength of the CGLN amplitudes between the s-wave resonances in the second resonance region and the resonances around 2.0 GeV in the quark model. There are two S_{11} resonances with isospin 1/2 in the second resonance region. The CGLN amplitudes for these two resonances are proportional to that in Eq. 58, in which the leading term does not depend on the outgoing meson momentum $\mathbf{q}$. On the other hand, the S or D wave resonances around 2 GeV belong to $n = 3$ in the harmonic oscillator basis. Acording to Eqs. 53 and 54, the amplitudes for the S and D wave resonances with $n = 3$ are at least proportional to $\mathbf{q}^2$ comparing to the $\mathbf{q}$ dependence of the amplitude of the $S_{11}(1535)$ in Eq. 58, as the wavefunctions for the S and D wave resonances with $n = 3$ are orthogonal to that of S_{11} resonances in the second resonance region. This leads to a smaller contributions from the S and D wave resonances around 2 GeV to the threshold region of the η' photoproductions.

Finally, the higher partial wave resonances become more important as the energy increases. Notice that the CGLN amplitudes for the P-wave resonances with $N = 2$, such as $P_{11}(1440)$ and $P_{11}(1710)$, are much smaller than those for the resonances $F_{15}(1680)$ and $F_{37}(1950)$. For the processes $\gamma N \to K\Sigma$, the contributions from the isospin 3/2 states, in particular those resonances in **56** multiplet, are dominant. Therefore, the processes $\gamma p \to K^+\Sigma^0$ and $\gamma p \to K^0\Sigma^+$ provide us a very important probe to the resonances with isospin 3/2, a particular example is the resonances $F_{37}(1950)$, $F_{35}(1905)$, $P_{33}(1920)$ and $P_{31}(1910)$. It should be pointed out that the F-wave resonances with isospin 3/2 were not included in most investigations. It raises the question whether these calculations are reliable beyond the threshold region.

5. Conclusion

A comprehensive and unified appraoch to the pseudo-scalar meson photoproductions is presented in this paper. The quark model approach represents a significant advance in the theory of the meson photoproductions. It introduces the quark and gluon degrees of freedom explicitly, which is an important step towards establishing the connection between the QCD and the reaction mechanism. It highlights the dynamic roles by the s-channel resonances, in particular the roles of the S_{11} resonances in the threshold region of the K, η and η' photoproductions.

Moreover, it should be pointed out that Eqs. 27 and 28 in the u-channel and Eqs. 50 and 51 in the s-channel are more general. They correspond to the pseudo scalar meson photoproductions at the quark level, which are

independent of the final states. Thus, if we replace the final nucleon and the Σ states by $\Delta(1232)$ and Σ^* states, the formulism presented here could be extended to the reactions $\gamma N \to \pi\Delta(1232)$ and $\gamma N \to K^+\Sigma^*$, which have not been investigated in the literature.

The initial results of our calculations in the η[17] and K[34] photoproductions has shown very good agreements between the theory and experimental data with far less parameters. The challenge for this approach would be to go one step further so that the quantitative descriptions of meson photoproductions, in particular the polarization observables that are sensitive to the detail structure of the s-channel resonances, could be provided. The numerical evaluations of the π and K photoproductions in this approach are currently in pregress, which will be reported elsewhere.

Acknowledgment

The financial support from Peking University is gratefully acknowledged.

Appendix

The matrix element for the nucleon pole term in the s-channel is found to be

$$\begin{aligned}\mathcal{M}_N = \omega_m e^{-\frac{\mathbf{q}^2+\mathbf{k}^2}{6\alpha^2}} \left(\frac{1}{E_f+M_f}+\frac{1}{E_N+M_N}\right)\left(e_N-\frac{\mathbf{k}^2}{4P_N\cdot k}\mu_N\right)\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon} \\ +ie^{-\frac{\mathbf{k}^2+\mathbf{q}^2}{6\alpha^2}}\left[\frac{\omega_m}{2}\left(\frac{1}{E_f+M_f}+\frac{1}{E_N+M_N}\right)+1\right]\frac{\mu_N}{2P_N\cdot k}\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot(\boldsymbol{\epsilon}\times\mathbf{k}) \\ +e^{-\frac{\mathbf{k}^2+\mathbf{q}^2}{6\alpha^2}}\left(\frac{1}{E_f+M_f}+\frac{1}{E_N+M_N}\right)\frac{e_N\omega_m}{4P_N\cdot k}\boldsymbol{\sigma}\cdot\mathbf{k}\boldsymbol{\epsilon}\cdot\mathbf{q} \\ +e^{-\frac{\mathbf{k}^2+\mathbf{q}^2}{6\alpha^2}}\left[\frac{\omega_m}{2}\left(\frac{1}{E_f+M_f}+\frac{1}{E_N+M_N}\right)+1\right]\frac{e_N}{2P_N\cdot k}\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\epsilon}\cdot\mathbf{q}\end{aligned} \quad (68)$$

where $P_N\cdot k=\omega_\gamma(E_N+\omega_\gamma)$, μ_N is the magnetic moments of the nucleon, e_N is the total charge of the nucleon.

The matrix elements for the U-channel Λ and Σ exchange term in the kaon production is

$$\begin{aligned}\mathcal{M}_{\Lambda\Sigma} = -e^{-\frac{\mathbf{k}^2+\mathbf{q}^2}{6\alpha^2}}\frac{M_f}{2M_N}\left(\frac{\mu_f}{P_f\cdot k}+\frac{g_{\Lambda\Sigma}\mu_{\Lambda\Sigma}}{P_S\cdot k\pm\delta m^2}\right) \\ \left\{\frac{\omega_m\mathbf{k}^2}{2}\left(\frac{1}{E_f+M_f}+\frac{1}{E_N+M_N}\right)\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}+\right.\end{aligned}$$

$$
i\left[\frac{\omega_m}{2}\left(\frac{1}{E_f+M_f}+\frac{1}{E_N+M_N}\right)+1\right]\boldsymbol{\sigma}\cdot(\boldsymbol{\epsilon}\times\mathbf{k})\boldsymbol{\sigma}\cdot\mathbf{q}\Big\}
$$
$$
-e^{-\frac{\mathbf{k}^2+\mathbf{q}^2}{6\alpha^2}}\left(\frac{1}{E_f+M_f}+\frac{1}{E_N+M_N}\right)\frac{e_f\omega_m}{4P_f\cdot k}\boldsymbol{\sigma}\cdot\mathbf{k}\boldsymbol{\epsilon}\cdot\mathbf{q}
$$
$$
-e^{-\frac{\mathbf{k}^2+\mathbf{q}^2}{6\alpha^2}}\left[\frac{\omega_m}{2}\left(\frac{1}{E_f+M_f}+\frac{1}{E_N+M_N}\right)+1\right]\frac{e_f}{2P_f\cdot k}\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\epsilon}\cdot\mathbf{q} \tag{69}
$$

where

$$
g_{\Lambda\Sigma}=\begin{cases}\frac{g_\Sigma}{g_\Lambda} & \text{for } \gamma N\to K\Lambda\\ \frac{g_\Lambda}{g_\Sigma} & \text{for } \gamma N\to K\Sigma^0\\ 0 & \text{other processes}\end{cases} \tag{70}
$$

is the ratio between the coupling constants for Λ and Σ final states, $\mu_{\Lambda\Sigma}=1.61$ is the magnetic moments for the transition between the Λ and Σ^0 states, and $P_f\cdot k=E_f\omega_\gamma+\mathbf{k}\cdot\mathbf{q}$, notice that the final baryon state has the total momentum $-\mathbf{q}$ in the center of mass system.

The u-channel nucleon exchange for the η and π productions is

$$
\mathcal{M}_u=-e^{\frac{\mathbf{k}^2+\mathbf{q}^2}{6\alpha^2}}\frac{\mu_f}{2P_f\cdot k}\Big\{\frac{\omega_m\mathbf{k}^2}{2}\left(\frac{1}{E_f+M_f}+\frac{1}{E_N+M_N}\right)\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}+
$$
$$
i\left[\frac{\omega_m}{2}\left(\frac{1}{E_f+M_f}+\frac{1}{E_N+M_N}\right)+1\right]\boldsymbol{\sigma}\cdot(\boldsymbol{\epsilon}\times\mathbf{k})\boldsymbol{\sigma}\cdot\mathbf{q}\Big\}
$$
$$
-e^{-\frac{\mathbf{k}^2+\mathbf{q}^2}{6\alpha^2}}\left(\frac{1}{E_f+M_f}+\frac{1}{E_N+M_N}\right)\frac{e_f\omega_m}{4P_f\cdot k}\boldsymbol{\sigma}\cdot\mathbf{k}\boldsymbol{\epsilon}\cdot\mathbf{q}
$$
$$
-e^{-\frac{\mathbf{k}^2+\mathbf{q}^2}{6\alpha^2}}\left[\frac{\omega_m}{2}\left(\frac{1}{E_f+M_f}+\frac{1}{E_N+M_N}\right)+1\right]\frac{e_f}{2P_f\cdot k}\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\epsilon}\cdot\mathbf{q} \tag{71}
$$

The matrix element for the t-channel is

$$
\mathcal{M}_t=e^{-\frac{(\mathbf{k}-\mathbf{q})^2}{6\alpha^2}}\frac{e_m(M_f+M_N)\mathbf{q}\cdot\boldsymbol{\epsilon}}{q\cdot k}\left(\frac{1}{E_f+M_f}\boldsymbol{\sigma}\cdot\mathbf{q}-\frac{1}{E_N+M_N}\boldsymbol{\sigma}\cdot\mathbf{k}\right) \tag{72}
$$

References

[1] M. Bockhorst, *et al*, Z. Phys. C**63**, 37(1994).

[2] J. Price it et al., Phys Rev. **C51**, R2283(1995).

[3] B. Krusche *et al*, Phys. Rev. Lett. **74**, 3736(1995).

[4] S. Dytman *et al.*, Phys. Rev. **C51**, 2170(1995).

[5] R. A. Shumacher, "Strangeness Electro- and Photo- Production at CEBAF", To be published in Few-Body Systems, Springer Verlag.

[6] R. A. Adelseck, C. Bennhold and L. E. Wright, Phys. Rev. **C32**, 1681(1985); H. Thom, Phys. Rev. **151**, 1322(1966).

[7] R. A. Adelseck and B. Saghai, Phys. Rev. **C42**, 108(1990).

[8] J.-C David, *et al.*, "Electromagnetic Production of Associated Strangeness", submitted to Phys. Rev. C.

[9] R. Williams, C. R. Ji, and S. Cotanch, Phys. Rev. **C46**, 1617(1992), *ibid*, **C43**, 452(1991), Phys. Rev. **D41**, (1990).

[10] C. Bennhold and H. Tanabe, Phys. Lett. **B243**, 12(1990).

[11] M. Benmerrouche, N. C. Mukhopadhyay, and J. F. Zhang, Phys. Rev. **D51**, 3237(1995). M. Benmerrouche and N. Mukhopadhyay, Phys. Rev. Lett. **67**, 101(1992).

[12] L. A. Copley, G. Karl and E. Obryk, Nucl. Phys. **B13**, 303(1969).

[13] R. P. Feynman, M. Kislinger and F. Ravndal, Phys. Rev. **D3**, 2706(1971).

[14] F. E. Close and Zhenping Li, Phys. Rev. **D42**, 2194(1990).

[15] Zhenping Li, Phys. Rev. **D50**, 5639(1994).

[16] Zhenping Li, Phys. Rev. **C52**, 1648(1995).

[17] Zhenping Li, Phys. Rev. **D52**, 4961(1995).

[18] A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189(1984).

[19] R. Koniuk and N. Isgur, Phys. Rev. **D21**, 1888(1980).

[20] G. F. Chew, M. L. Goldberger, F. E. Low and Y. Nambu, Phys. Rev. **106**, 1345(1957); S. Fubini, G. Furlan and C. Rossetti, Nuovo Cimento, **40**, 1171(1965).

[21] C. G. Fasano, F. Tabakin and B. Saghai, Phys. Rev. **C46**, 2430(1992).

[22] I thank C. Bennhold who pointed this out to me.

[23] R. Dolen, D. Horn, and C. Schmid, Phys. Rev. **166**, 1768(1966).

[24] C.A. Dominquez, *Rev. del Nuovo Cimento*, **8**, 1(1985).

[25] B. R. Holstein, Baryon 92, edited by Moshe Gai, World Scientific, (1993).

[26] Zhenping Li, Phys. Rev. **D48**, 3070(1993).

[27] For the pion-photoproduction, see the summary paper by D. Drechsel and L. Tiator, J. Phys. G: Nucl. Part.

[28] J. M. Blatt and V. F. Weisskopf, Theoretical Nuclear Physics, (Wiley, New York, 1952), p361.

[29] Zhenping Li, M. Guidry, T. Barnes and E. S. Swanson, MIT-ORNL preprint, MIT-CTP-2277/ORNL-CCIP-94-01.

[30] Zhenping Li and F.E. Close, Phys. Rev. **D42**, 2207(1990).

[31] R. G. Moorhouse, Phys. Rev. Lett. **16**, 772(1966).

[32] Zhenping Li and R. Workman "Do We have Three Resonances In the Second Resonance Region?", to appear on Phys. Rev. C.

[33] Zhenping Li, "The η' photoproduction off nucelons in the quark model", nucl-th/9607043.

[34] Zhenping Li, Wei-Hsing Ma, and Zhang Lin, Phys. Rev. **C54**, R2171(1996).

[35] B. Ritchie *et al.*, CEBAF proposal, (unpublished).

Table 1: The g-factors in the u-channel amplitudes in Eqs. 31 and 32 for different production processes.

Reaction	g_3^u	g_2^u	g_v	g_v'	g_a'	g_A	g_S
$\gamma p \rightarrow K^+\Lambda$	$-\frac{1}{3}$	$\frac{1}{3}$	1	1	1	$\sqrt{\frac{3}{2}}$	$-\frac{\mu_\Lambda}{3}$
$\gamma n \rightarrow K^0\Lambda$	$-\frac{1}{3}$	$\frac{1}{3}$	1	-1	-1	$\sqrt{\frac{3}{2}}$	$\frac{\mu_\Lambda}{3}$
$\gamma p \rightarrow K^+\Sigma^0$	$-\frac{1}{3}$	$\frac{1}{3}$	-3	-7	9	$-\frac{1}{3\sqrt{2}}$	μ_{Σ^0}
$\gamma n \rightarrow K^0\Sigma^0$	$-\frac{1}{3}$	$\frac{1}{3}$	-3	11	-9	$\frac{1}{3\sqrt{2}}$	μ_{Σ^0}
$\gamma p \rightarrow K^0\Sigma^+$	$-\frac{1}{3}$	$\frac{4}{3}$	-3	2	0	$\frac{1}{3}$	$\frac{2\mu_{\Sigma^+}}{3}$
$\gamma n \rightarrow K^+\Sigma^-$	$-\frac{1}{3}$	$-\frac{2}{3}$	-3	2	0	$-\frac{1}{3}$	0
$\gamma p \rightarrow \eta p$	1	0	1	0	0	1	0
$\gamma n \rightarrow \eta n$	$-\frac{2}{3}$	$\frac{2}{3}$	0	-1	0	1	0
$\gamma p \rightarrow \pi^+ n$	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{3}{5}$	$\frac{1}{5}$	$\frac{9}{5}$	$\frac{5}{3}$	$-\frac{2\mu_n}{5}$
$\gamma n \rightarrow \pi^- p$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{3}{5}$	$\frac{1}{5}$	$-\frac{9}{5}$	$-\frac{5}{3}$	$-\frac{4\mu_p}{15}$
$\gamma p \rightarrow \pi^0 p$	$\frac{7}{15}$	$\frac{8}{15}$	$\frac{15}{7}$	2	0	$\frac{5}{3\sqrt{2}}$	$\frac{8\mu_p}{15}$
$\gamma n \rightarrow \pi^0 n$	$-\frac{2}{15}$	$\frac{2}{15}$	6	-7	0	$-\frac{5}{3\sqrt{2}}$	$\frac{4\mu_n}{5}$

Tbale 2: Meson transition amplitudes A in the simple harmonic oscillator basis.

(N, L)	Partial Waves	A
$(0,0)$	P	$-\left(\frac{\omega_m}{E_f+M_f}+1\right)$
$(1,1)$	S	$\frac{\omega_m}{\mu_q}-\left(\frac{\omega_m}{E_f+M_f}+1\right)\frac{2\mathbf{q}^2}{3\alpha^2}$
$(1,1)$	D	$-\left(\frac{\omega_m}{E_f+M_f}+1\right)$
$(2,0)$	P	$\frac{\omega_m}{\mu_q}-\left(\frac{\omega_m}{E_f+M_f}+1\right)\frac{\mathbf{q}^2}{\alpha^2}$
$(2,2)$	P	$\frac{\omega_m}{\mu_q}-\left(\frac{\omega_m}{E_f+M_f}+1\right)\frac{2\mathbf{q}^2}{5\alpha^2}$
$(2,2)$	F	$-\left(\frac{\omega_m}{E_f+M_f}+1\right)\frac{\mathbf{q}^2}{\alpha^2}$

Table 5: The g_R-factors in the s-channel resonance amplitudes for different production processes.

Reaction	$(56,^2 N)$	$(56,^4 \Delta)$	$(70,^2 N)$	$(70,^4 N)$	$(70,^2 \Delta)$
$\gamma p \to K^+\Lambda$	1	0	1	0	0
$\gamma n \to K^0\Lambda$	1	0	1	0	0
$\gamma p \to K^+\Sigma^0$	1	$\frac{8}{3}$	-1	0	1
$\gamma n \to K^0\Sigma^0$	1	-$\frac{8}{3}$	-1	-2	-1
$\gamma p \to K^0\Sigma^+$	1	-$\frac{4}{3}$	-1	0	-$\frac{1}{2}$
$\gamma n \to K^+\Sigma^-$	1	$\frac{4}{3}$	-1	-2	$\frac{1}{2}$
$\gamma p \to \eta p$	1	0	2	0	0
$\gamma n \to \eta n$	1	0	2	1	0
$\gamma p \to \pi^+ n$	1	$\frac{4}{15}$	$\frac{4}{5}$	0	$\frac{1}{10}$
$\gamma n \to \pi^- p$	1	-$\frac{4}{15}$	$\frac{4}{5}$	-$\frac{1}{5}$	-$\frac{1}{10}$
$\gamma p \to \pi^0 p$	1	-$\frac{8}{15}$	$\frac{4}{5}$	0	$\frac{1}{10}$
$\gamma n \to \pi^0 n$	1	$\frac{8}{15}$	$\frac{4}{5}$	-$\frac{1}{5}$	-$\frac{1}{10}$

Table 3: The CGLN amplitudes for the S-channel baryons resonances for the proton target in the $SU(6)\otimes O(3)$ symmetry limit, where $k=|\mathbf{k}|$, $q=|\mathbf{q}|$, and $x=\frac{\mathbf{k}\cdot\mathbf{q}}{kq}$. The CGLN amplitudes for the $N(^4P_M)$, $N(^4S_M)$, and $N(^4D_M)$ states are zero due to the Moorhouse selection rule, see text.

States	f_1	f_2	f_3	f_4
$\Delta(^4S_s)\frac{3}{2}^+$	$3\frac{kqx}{2m_q}$	$2\frac{1}{2m_q}$	$-3\frac{1}{2m_q}$	0
$N(^2P_M)\frac{1}{2}^-$	$\frac{\omega_\gamma}{12}\left(1+\frac{k}{2m_q}\right)$	0	0	0
$N(^2P_M)\frac{3}{2}^-$	$-\frac{\omega_\gamma}{18}\left(1+\frac{k}{2m_q}\right)\frac{q^2}{\alpha^2}$	$-\frac{kqx}{12m_q\alpha^2}$	0	$\frac{\omega_\gamma}{6\alpha^2}$
$\Delta(^2P_M)\frac{1}{2}^-$	$\frac{\omega_\gamma}{6}\left(1-\frac{k}{6m_q}\right)$	0	0	0
$\Delta(^2P_M)\frac{3}{2}^-$	$-\frac{\omega_\gamma}{9}\left(1-\frac{k}{6m_q}\right)\frac{q^2}{\alpha^2}$	$\frac{kqx}{18m_q\alpha^2}$	0	$\frac{\omega_\gamma}{3\alpha^2}$
$N(^2S_s')\frac{1}{2}^+$	0	$-\frac{k^2}{216m_q\alpha^2}$	0	0
$\Delta(^4S_s')\frac{3}{2}^+$	$\frac{k^3qx}{36m_q\alpha^2}$	$\frac{k^2}{54m_q\alpha^2}$	$-\frac{k^2}{36m_q\alpha^2}$	
$N(^2D_s)\frac{3}{2}^+$	$\frac{k^2qx}{36\alpha^2}\left(1+\frac{k}{2m_q}\right)$	$\frac{k^2}{216m_q\alpha^2}$	$\frac{\omega_\gamma}{36\alpha^2}$	0
$N(^2D_s)\frac{5}{2}^+$	$-\frac{k^2qx}{180\alpha^2}\left(1+\frac{k}{2m_q}\right)$	$-\frac{k^2}{144m_q\alpha^2}\left(x^2-\frac{1}{5}\right)$	$-\frac{k}{180\alpha^2}$	$\frac{k^2x}{36q\alpha^2}$
$\Delta(^4D_s)\frac{1}{2}^+$	0	$-\frac{k^2}{108m_q\alpha^2}$	0	0
$\Delta(^4D_s)\frac{3}{2}^+$	0	$-\frac{k^2}{108m_q\alpha^2}$	$\frac{k^2}{36m_q\alpha^2}$	0
$\Delta(^4D_s)\frac{5}{2}^+$	$\frac{k^2}{126m_q\alpha^2}$	$\frac{k^2}{126m_q\alpha^2}\left(x^2-\frac{1}{5}\right)$	$\frac{k^2}{210m_q\alpha^2}$	0
$\Delta(^4D_s)\frac{7}{2}^+$	$\frac{k^2}{12m_q\alpha^2}\left(x^2-\frac{3}{7}\right)$	$\frac{k^2}{21m_q\alpha^2}\left(x^2-\frac{1}{5}\right)$	$\frac{k^2}{12m_q\alpha^2}\left(x^2-\frac{1}{7}\right)$	0
$N(^2S_M)\frac{1}{2}^+$	0	$-\frac{k^2}{432m_q\alpha^2}$	0	0
$\Delta(^2S_M)\frac{1}{2}^+$	0	$-\frac{k^2}{648m_q\alpha^2}$	0	0
$N(^2D_M)\frac{3}{2}^+$	$\frac{k^2qx}{72\alpha^2}\left(1+\frac{k}{2m_q}\right)$	$\frac{k^2}{432m_q\alpha^2}$	$\frac{k}{72\alpha^2}$	0
$N(^2D_M)\frac{5}{2}^+$	$\frac{-k^2qx}{360\alpha^2}\left(1+\frac{k}{2m_q}\right)$	$\frac{-k^2}{288m_q\alpha^2}\left(x^2-\frac{1}{5}\right)$	$\frac{-k}{360\alpha^2}$	$\frac{k^2x}{72q\alpha^2}$
$\Delta(^2D_M)\frac{3}{2}^+$	$\frac{k^2qx}{36\alpha^2}\left(1+\frac{k}{2m_q}\right)$	$\frac{-k^2}{648m_q\alpha^2}$	$\frac{k}{36\alpha^2}$	0
$\Delta(^2D_M)\frac{5}{2}^+$	$\frac{-k^2qx}{180\alpha^2}\left(1+\frac{k}{2m_q}\right)$	$\frac{k^2}{432m_q\alpha^2}\left(x^2-\frac{1}{5}\right)$	$\frac{-k}{180\alpha^2}$	$\frac{k^2x}{36q\alpha^2}$

Table 4: The CGLN amplitudes for the S-channel baryons resonances for the neutron target in the $SU(6) \otimes O(3)$ symmetry limit, where $k = |\mathbf{k}|$, $q = |\mathbf{q}|$, and $x = \frac{\mathbf{k}\cdot\mathbf{q}}{kq}$.

States	f_1	f_2	f_3	f_4
$N(^2P_M)\frac{1}{2}^-$	$\frac{-\omega_\gamma}{12}\left(1+\frac{k}{6m_q}\right)$	0	0	0
$N(^2P_M)\frac{3}{2}^-$	$\frac{\omega_\gamma}{18}\left(1+\frac{k}{6m_q}\right)\frac{q^2}{\alpha^2}$	$\frac{kqx}{36m_q\alpha^2}$	0	$\frac{-\omega_\gamma}{6\alpha^2}$
$N(^4P_M)\frac{1}{2}^-$	$\frac{-\omega_\gamma k}{36m_q}$	0	0	0
$N(^4P_M)\frac{3}{2}^-$	$\frac{-\omega_\gamma kq^2}{135m_q\alpha^2}$	$\frac{-kqx}{90m_q\alpha^2}$	0	0
$N(^4P_M)\frac{5}{2}^-$	$\frac{-\omega_\gamma kq^2}{6m_q\alpha^2}\left(x^2-\frac{1}{5}\right)$	$\frac{-kqx}{10m_q\alpha^2}$	$\frac{kqx}{6m_q\alpha^2}$	0
$N(^2S'_s)\frac{1}{2}^+$	0	$\frac{k^2}{324m_q\alpha^2}$	0	0
$N(^2D_s)\frac{3}{2}^+$	$\frac{-k^3qx}{108m_q\alpha^2}$	$\frac{-k^2}{324m_q\alpha^2}$	0	0
$N(^2D_s)\frac{5}{2}^+$	$\frac{k^3qx}{540m_q\alpha^2}$	$\frac{k^2}{216m_q\alpha^2}\left(x^2-\frac{1}{5}\right)$	0	0
$N(^2S_M)\frac{1}{2}^+$	0	$\frac{k^2}{1296m_q\alpha^2}$	0	0
$N(^2D_M)\frac{3}{2}^+$	$\frac{-k^2qx}{72\alpha^2}\left(1+\frac{k}{6m_q}\right)$	$\frac{-k^2}{1296m_q\alpha^2}$	$\frac{-k}{72\alpha^2}$	0
$N(^2D_M)\frac{5}{2}^+$	$\frac{k^2qx}{360\alpha^2}\left(1+\frac{k}{6m_q}\right)$	$\frac{k^2}{864m_q\alpha^2}\left(x^2-\frac{1}{5}\right)$	$\frac{k}{360\alpha^2}$	$\frac{-k^2x}{72q\alpha^2}$
$N(^4S_M)\frac{3}{2}^+$	$\frac{-k^3qx}{216m_q\alpha^2}$	$\frac{-k^2}{324m_q\alpha^2}$	$\frac{k^2}{216m_q\alpha^2}$	0
$N(^4D_M)\frac{1}{2}^+$	0	$\frac{-k^2}{1944m_q\alpha^2}$	0	0
$N(^4D_M)\frac{3}{2}^+$	$\frac{k^3qx}{162m_q\alpha^2}$	$\frac{7k^2}{1944m_q\alpha^2}$	$\frac{-k^2}{216m_q\alpha^2}$	0
$N(^4D_M)\frac{5}{2}^+$	$\frac{-k^3qx}{756m_q\alpha^2}$	$\frac{-k^2}{756m_q\alpha^2}\left(x^2-\frac{1}{5}\right)$	$\frac{k^2}{1260m_q\alpha^2}$	0
$N(^4D_M)\frac{7}{2}^+$	$\frac{-k^3q}{72m_q\alpha^2}\left(x^2-\frac{3}{7}\right)$	$\frac{-k^2}{126m_q\alpha^2}\left(x^2-\frac{1}{5}\right)$	$\frac{k^2}{72m_q\alpha^2}\left(x^2-\frac{1}{7}\right)$	0